GWR（裾切りガウスの問題解決）

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1　GWR ··· 推定方法

$$W_i y = W_i X \beta_i + \epsilon_i \tag{1}$$

$$\left[\begin{array}{cccc} h_{i1}y_1 & h_{i2}y_2 & \cdots & h_{in}y_n \end{array}\right]^T = \tag{2}$$

$$\left[\begin{array}{cccc} \beta_{01}h_{i1}+\beta_{11}h_{i1}x_1 & \beta_{02}h_{i2}+\beta_{12}h_{i2}x_2 & \cdots & \beta_{0n}h_{in}+\beta_{1n}h_{in}x_n \end{array}\right]^T + \left[\begin{array}{cccc} \epsilon_1 & \epsilon_2 & \cdots & \epsilon_n \end{array}\right] \tag{3}$$

ここで、最小二乗法より（地区 i の推定量を求めるとき）

$$Q = \sum_{k=1}^{n} \epsilon_k^2 = \sum_{k=1}^{n} h_{ki}^2 y_k - \beta_{0i} - \beta_{1i} x_k{}^2$$

$$\begin{aligned} \frac{\partial Q}{\partial \beta_{0i}} &= -2\sum_{k=1}^{n} h_{ki}^2 \{y_k - \beta_{0i} - \beta_{1i}x_k\} = 0 \\ \frac{\partial Q}{\partial \beta_{1i}} &= -2\sum_{k=1}^{n} h_{ki}^2 \{y_k - \beta_{0i} - \beta_{1i}x_k\} x_k = 0 \end{aligned}$$

整理して

$$\sum_{k=1}^{n} h_{ki}^2 \{y_k - \beta_{0i} - \beta_{1i}x_k\} = 0$$
$$\sum_{k=1}^{n} h_{ki}^2 \{y_k - \beta_{0i} - \beta_{1i}x_k\} x_k = 0$$

$$\left(\sum_{k=1}^{n} h_{ki}^2\right)\beta_{0i} + \left(\sum_{k=1}^{n} h_{ki}^2 x_k\right)\beta_{1i} = \left(\sum_{k=1}^{n} h_{ki}^2 y_k\right)$$
$$\left(\sum_{k=1}^{n} h_{ki}^2 x_k\right)\beta_{0i} + \left(\sum_{k=1}^{n} h_{ki}^2 x_k^2\right)\beta_{1i} = \left(\sum_{k=1}^{n} h_{ki}^2 x_k y_k\right)$$

$$\begin{bmatrix} \sum_{k=1}^{n} h_{ki}^2 & \sum_{k=1}^{n} h_{ki}^2 x_k \\ \sum_{k=1}^{n} h_{ki}^2 x_k & \sum_{k=1}^{n} h_{ki}^2 x_k^2 \end{bmatrix} \begin{bmatrix} \beta_{0i} \\ \beta_{1i} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{n} h_{ki}^2 y_k \\ \sum_{k=1}^{n} h_{ki}^2 x_k y_k \end{bmatrix} \tag{4}$$

つまり、

$$(X^T W_i^2 X)\beta_i = (X^T W_i^2 y)$$

$$\hat{\beta}_i = (X^T W_i^2 X)^{-1}(X^T W_i^2 y)$$